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PERIODIC WAKES OF LOW ASPECT RATIO CYLINDERS WITH FREE HEMISPHERICAL ENDS

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The wakes of circular cylinders with free hemispherical ends and of different aspect ratios (length-to-diameter ratios) are experimentally studied for moderate Reynolds numbers. This investigation is restricted to cylinders with low aspect ratios, namely less than 5, and includes the case of the sphere. The transition to nonstationarity of the flow in these cylinder wakes is the main focus of this work: the results show that the stability of wakes is strongly dependent on aspect ratio and is also affected by the free-end conditions. We characterize the frequency, amplitude and phase as well as the critical Reynolds number of the periodic vortex shedding regime.

1. INTRODUCTION

THE ALTERNATE VORTEX SHEDDING from the two sides of a cylinder, due to the Bénard–von Kármán instability, has received a great deal of attention in the past and these researches have been the object of extensive reviews [e.g., Williamson (1996)]. Although three-dimensional phenomena, such as oblique shedding, cellular structure and dislocations, are dominant for finite cylinders, even with large length-to-diameter ratios, the studies deal mainly with the two-dimensional parallel shedding. On the one hand, recent investigations have been in part devoted to experimentally promote parallel vortex shedding by manipulation of the end conditions [see, e.g., Williamson (1988), Eisenlohr & Eckelmann (1989), Hammache & Gharib (1989)] and, on the other hand, to model the three-dimensional structure of the cylinder wake. For example, Albarède & Monkewitz (1992) and Albarède & Provansal (1995) have shown the ability of the phenomenological Ginzburg–Landau model to describe the three-dimensional phenomena observed in the wake of a finite circular cylinder as a collective interaction of nonlinear oscillators.

For a finite cylinder with "natural" end conditions (i.e., without manipulation), namely with end plates parallel to the free stream or with free ends, experiments of Gerich & Eckelmann (1982) show a cellular structure for the periodic wake, with cells of lower frequency in the regions near the ends, for long enough cylinders [see also Williamson (1989)]. Free-end cylinder wakes have also been investigated numerically by Dauchy *et al.* (1997) and experimentally by Slaouti & Gerrard (1981); they report that the vortex shedding is diminished or suppressed near the free ends.

The sphere appears as the limiting case of the geometry of cylinders with two free hemispherical ends as considered in the present investigation. In contrast to the cylinder, for the sphere wake, the transition to a time-dependent (periodic) flow is preceded by a regular axisymmetry-breaking bifurcation giving rise to a wake with a plane symmetry, as described

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e.g. in recent experimental (Sakamoto & Haniu 1995; Ormières 1999) or numerical (Johnson & Patel 1999; Tomboulides & Orszag 2000; Ghidersa & Dušek 2000) studies. Although this plane symmetry is maintained in the periodic regime resulting from this transition [Mittal (1999), and the previously cited references], the visualizations [see, e.g., Leweke *et al.* (1999)] of the sphere wake reveals a complex three-dimensional spatial structure with the periodic shedding of connected vortex loops.

Zdravkovich *et al.* (1989) have performed experiments for low aspect ratio free-end cylinders. In particular, using a oil-film visualization technique, they report a change of the symmetry of the surface pattern when the aspect ratio is reduced below 3.

In this paper, the wakes of circular cylinders with free hemispherical ends, including the sphere, are experimentally studied at moderate Reynolds number. The experimental set-up is presented in Section 2. Effects of aspect ratio and free ends on the onset of transition to the periodic wake are discussed in Section 3. The periodic state is characterized in Section 4 and the conclusion and the perspectives are given in Section 5.

2. EXPERIMENTAL DETAILS

The wake experiments were carried out in a square test-section ($0{\cdot}25 \times 0{\cdot}25\ \mathrm{m}^2$) of an open low-turbulence wind tunnel. Nine circular cylinders of same diameter $D = 10$ mm and of different lengths L were used. To assure a "continuous" change of the body geometry from the cylinder to the sphere, hemispherical ends were used. The values of the aspect ratio L/D were 5, 4, 3, 2·6, 2·3, 2, 1·6, 1·3 and 1, respectively; the case $L/D = 1$ corresponding to the sphere. The cylinders were mounted horizontally at the centre of the working section, with axis perpendicular to the free stream. They were held in their centre by a bent rigid thin rod, as shown in Figure 1, in such a way that their ends were free. The rod was upstream of the cylinders and inclined with the free stream of an angle α of order $10°$. One effect of this holding system was to induce a weak velocity gradient and thus to freeze the symmetry plane (see Section 1) of the sphere wake in the plane containing the rod (Sakamoto & Haniu 1995; Ormières & Provansal 1999). Ormières (1999) has shown that, for the sphere, the upstream bent rod introduces no change to the dynamics of the wake but can slightly vary its threshold of transition to nonstationarity.

The velocities were measured by laser Doppler anemometry with a mobile measurement point. The free-stream velocity U was measured $6D$ upstream of the cylinders and corrections were applied for the blockage effect and for the growth of the boundary layers on the

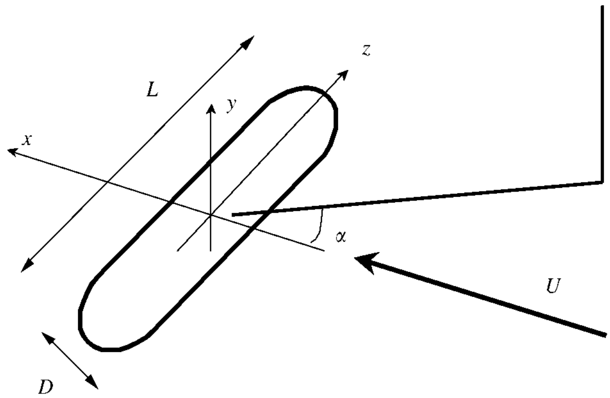

Figure 1. Experimental geometry and coordinate system.

tunnel walls. The temperature of the flow was measured with a thermocouple to calculate the kinematic viscosity ν of air. Frequencies f and amplitudes of the vortex shedding modes were obtained by spectral analysis of the streamwise velocity measured in the wakes. For the phase measurements, a hot-wire anemometer was used. The hot-wire sensor was placed at a fixed position in the wake of the cylinders and the average phase difference relative to the signal of the mobile laser Doppler anemometer was deduced from the cross-correlation function.

For a given cylinder, the wake dynamics is controlled by the Reynolds number based on the cylinder diameter $\mathrm{Re} = UD/\nu$ which was varied between 50 and 400 during the present experiments. The shedding frequencies f are expressed in nondimensional form either with the Strouhal number $\mathrm{St} = fD/U$ or with the Roshko number $\mathrm{Ro} = fD^2/\nu$ which is based on the diffusion time. The interest in using Ro is to avoid the free-stream velocity U which has the highest associated uncertainty in the experiments. The estimated errors for the different quantities give a statistical error for Re and for Ro in the periodic regime of less than 2%.

For the following, we use the Cartesian coordinates (x, y, z) defined in Figure 1, with the origin at the centre of the cylinder.

3. TRANSITION TO NONSTATIONARITY

Increasing the Reynolds number, at a critical value Re_c, we observe a transition from a stationary state to a time-dependent regime for the flow in the wake of a cylinder. Figure 2 shows the evolution with the Reynolds number Re of the square of the amplitude of the streamwise velocity fluctuation u'_x measured for the sphere $(L/D = 1)$ at fixed positions $(x/D = 6{\cdot}5,\ y/D = -0{\cdot}5,\ z/D = 0{\cdot}5)$ in the wake. At a well-defined value Re_c of Re, the fluctuation amplitude deviates from zero; this critical Reynolds number for the transition to the nonstationarity is then determined by extrapolation to zero amplitude of the linear behaviour of the $u'^2_x(\mathrm{Re})$ relationship near the threshold Re_c. Such a linear behaviour is consistent with a Landau–Hopf bifurcation. The critical value Re_c for a given cylinder has been found to be independent of the location of the measurement point. These fluctuation amplitude measurements have been performed for both increasing and decreasing the Reynolds number and no evidence of a hysteretic cycle has been detected. Thus, the bifurcation associated with the appearance of the nonstationary wake flow appears supercritical for all the cylinders considered.

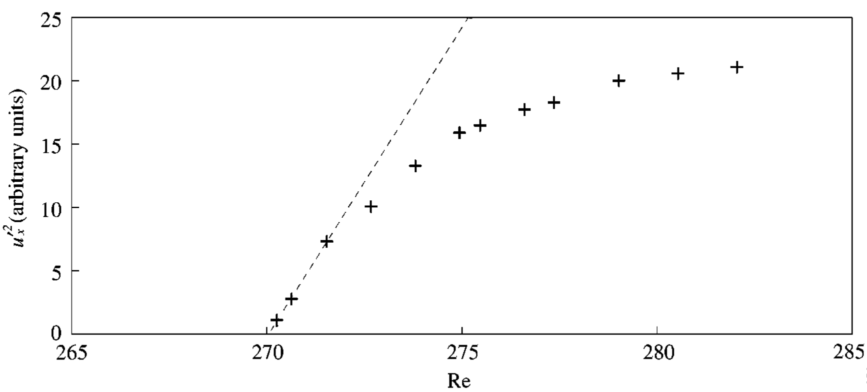

Figure 2. Square of the amplitude of the streamwise velocity fluctuation as a function of Reynolds number. $L/D = 1$ (sphere), $x/D = 6{\cdot}5$, $y/D = -0{\cdot}5$, $z/D = 0{\cdot}5$.

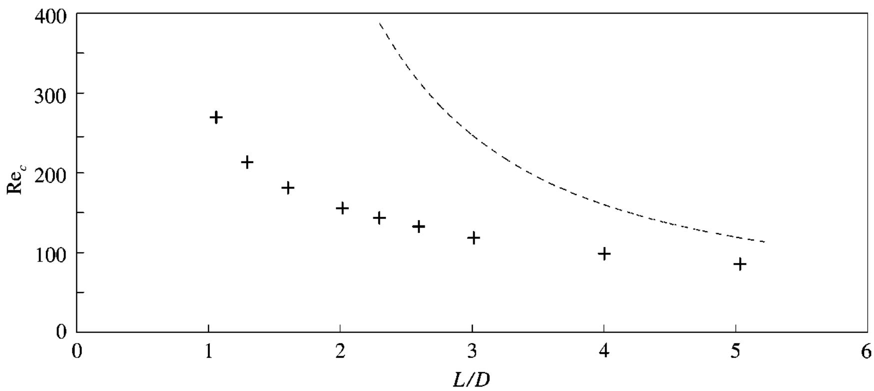

Figure 3. Critical Reynolds number as a function of aspect ratio; - - -, cylinder confined between two end plates (Norberg 1994).

The critical values Re_c we have obtained for the different cylinders are displayed in Figure 3 as a function of the aspect ratio L/D. This diagram shows the strong influence of the aspect ratio on the stability of the wake flow: the smaller the L/D, the more stable is the stationary regime. In fact, the value of Re_c evolves from about 85 for $L/D = 5$ to about 270 for $L/D = 1$. The critical Reynolds number we obtained for the sphere (of about 270) agrees with the values found in the recent numerical simulations of Johnson & Patel (1999), Tomboulides & Orszag (2000) and Ghidersa & Dušek (2000) which are between 270 and 280. The small difference with the critical value given in the experimental study of Ormières & Provansal (1999), between 275 and 285, can be explained by a few experimental details. In our case, the sphere, constructed from two hemispheres, presents a slight deviation from the perfect sphere used by Ormières & Provansal; moreover, the upstream rod tends to weakly decrease this value.

The stabilizing effect of the reduction of the aspect ratio has been also observed for cylinders confined between two end plates parallel with the free stream (Mathis *et al.* 1984; Lee & Budwig 1991; Norberg 1994). The results obtained by Norberg for aspect ratio larger than 2 are displayed in Figure 3. These observations agree, at least qualitatively, with the Ginzburg–Landau model which predicts that Re_c is given by the relation $\mathrm{Re}_c = \mathrm{Re}_\infty + B(L/D)^{-2}$, where Re_∞ is the critical Reynolds number for the parallel vortex shedding ($L/D \to \infty$) which is known to be about 47; the constant B is proportional to the diffusive coupling constant μ_r of the model [see, e.g., Albarède & Provansal (1995)].

Let us note that Albarède & Provansal (1995) have specified that the characteristic length to be considered in the model is not the geometrical one of the cylinder L but an effective length L' of the oscillating modes (see the next section). Finally, the comparison with the results of Norberg (1994) of the critical values Re_c (Figure 3) shows that the free-end conditions tend to decrease the stability of the cylinder wake. This could be explained by a smaller effective length due to the three-dimensional flow near the free ends.

4. PERIODIC REGIME

For Reynolds numbers larger than the critical value Re_c, the spectrum of the streamwise velocity fluctuations measured in the wake, presents a well-defined peak at a frequency f, and eventually at its harmonics, characteristic of a periodic state. This single-frequency regime, corresponding to periodic vortex shedding, continues up to a second critical value

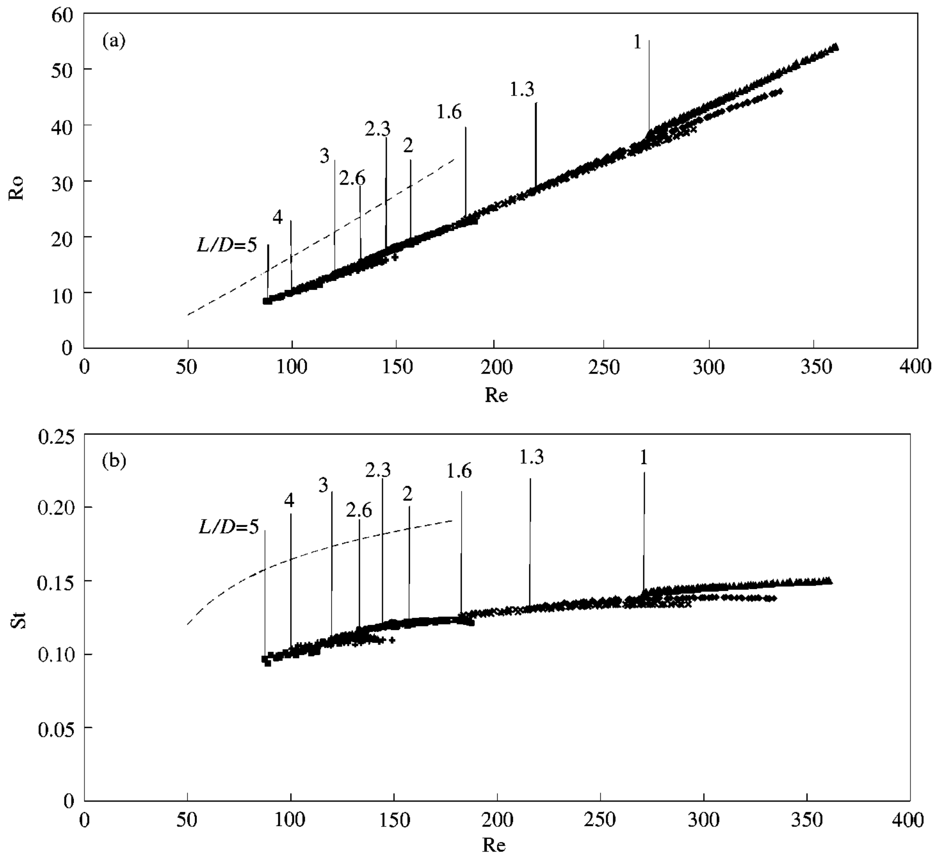

Figure 4. (a) Roshko number and (b) Strouhal number as functions of Reynolds number for the different aspect ratios; - - -, parallel shedding (Leweke & Provansal 1995). The vertical lines mark the thresholds of transition to the periodic regime for the nine cylinders.

Re_{c2} of the Reynolds number (also a function of aspect ratio) after which the velocity fluctuations appear less regular and their spectrum is characterized by the presence of a second peak at a lower frequency. When the Reynolds number is further increased, nonlinear interactions between modes contribute to the appearance of other peaks at frequencies equal to linear combinations of the two frequencies.

To characterize the periodic regime of cylinder wakes, the frequency of the vortex shedding has been determined. First, we can note that the same frequency has been found for different locations in the wake. In particular, no variation of the frequency along the span of the cylinder has been detected, meaning that the wake has a one-cell structure. This observation is consistent with the experimental results of Gerich & Eckelmann (1982), who report that the cellular structure of the wake disappears when the cylinder length is shorter than the dimension of the two end cells.

Figure 4(a,b) presents the dimensionless shedding frequency, expressed respectively, in terms of the Roshko number Ro and the Strouhal number St. The variation of Ro appears as an increasing function of the Reynolds number and is quite continuous for all the considered cylinders but with a slight change of the slope for the sphere ($L/D = 1$). For the sphere wake at a Reynolds number of 300, the numerical studies of Johnson & Patel (1999), Tomboulides & Orszag (2000) and Ghidersa & Dušek (2000) furnish values for Ro between

40·5 and 41·1, which are slightly below the value of about 43·4 we found. According to the experimental study of Sakamoto & Haniu (1995) of a sphere in a uniform shear flow, this difference could be due to the velocity gradient induced by the upstream rod (see Section 2).

Although the upper limits Re_{c2} have not been precisely determined, we can also note the broadening of the Reynolds number range [Re_c, Re_{c2}] for the periodic regime when the aspect ratio is decreased. This stabilizing effect of the reduction of the aspect ratio upon the limit Re_{c2} is consistent with the evolution of the critical Reynolds number Re_c.

For comparison, we have displayed in Figure 4 the frequencies obtained by Leweke & Provansal (1995) for parallel shedding at Reynolds numbers between 47 and 180. We note that the shedding frequencies in the wake of low aspect ratio cylinders with free hemispherical ends are much smaller than the frequencies associated with the parallel shedding. In particular, in the whole Reynolds number range considered, the Strouhal number is less than the asymptotic value of 0.2 for parallel shedding. A similar observation has been reported by Gerich & Eckelmann (1982) for cylinders confined between two parallel end plates when the cylinder length is smaller than the dimension of the two lower frequency end cells. The difference of frequencies between oblique and parallel shedding has been explained by Williamson (1988) as a consequence of the inclination θ of the vortex lines leading to a universal St(Re) curve after correction for the $\cos\theta$ term. For example, at $\mathrm{Re} = 92$ (value corresponding to the measurements of Figure 5), we measure $\mathrm{Ro} \approx 8{\cdot}9$ for the cylinder of aspect ratio $L/D = 5$, which is compared with the value of 14·7 deduced from the universal parallel shedding law giving an average inclination angle of about 52° ($\cos\theta = 8{\cdot}9/14.7$).

In Figure 5(a), we present an example (for $L/D = 5$, $\mathrm{Re} = 92$) of the amplitude evolution of the streamwise velocity fluctuation with the spanwise position z. We can see that this amplitude is strongly diminished near the free ends of the cylinder as reported in the numerical work of Dauchy *et al.* (1997). We also note that, from flow visualizations, Slaouti & Gerrard (1981) state that the free-end effect is to suppress the vortex shedding in the vicinity of the ends.

Thus, one can observe a significant amplitude in the spanwise direction for z in the interval $[-1D, +1D]$. The effective length L' of the oscillating mode is then determined by intersecting the cosine law fit [dashed curve in Figure 5(a)], characteristic of a single-mode regime, with the z-axis. This length L' is much smaller than the geometric length L of the cylinder which extends from $-2{\cdot}5D$ to $2{\cdot}5D$. The subcritical flow near the boundary region of the ends suppresses the instability and reduces the effective length L'. Spanwise evolution measurements of the fluctuation amplitude have been repeated for all the cylinders at different Reynolds number. The effective length L' appears to be function not only of the aspect ratio but also of the Reynolds number.

The isophase displayed in Figure 5(b) has been obtained under the same conditions ($L/D = 5$ and $\mathrm{Re} = 92$) as the amplitude measurements [Figure 5(a)]. This curve is deduced from measurements of the spanwise and streamwise phase evolutions by neglecting the variation of the phase with the transverse coordinate y. The phase varies linearly in the streamwise direction with a spatial wavelength which evolves from $5{\cdot}7D$ to $7{\cdot}5D$ for cylinders of aspect ratio from 1 to 5, respectively. The isophase curve presents a flat hat shape. From this plot, the angle of inclination of the vortices linked to the strong variation at the sides would be about 70°, which is in agreement with the average value deduced from the frequency measurements.

The effective length L' has been determined for Reynolds numbers close to the threshold Re_c for the cylinders of aspect ratio 5, 4, 3 and 2. The corresponding values of Re_c are displayed in Figure 6 as function of $(L'/D)^{-2}$ for comparison with the Ginzburg–Landau model.

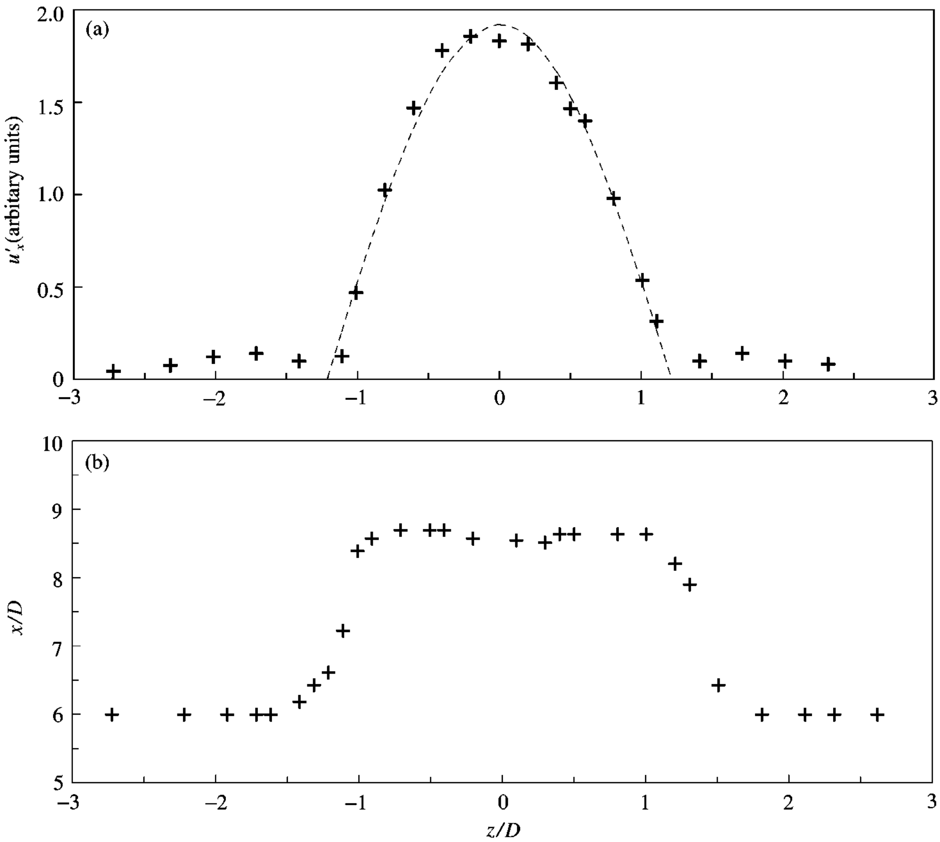

Figure 5. (a) Amplitude of the streamwise velocity fluctuation as a function of spanwise position; - - -, fit with a cosine law ($L/D = 5$, Re = 92; $x/D = 6$, $y/D = -1{\cdot}25$). (b) Isophase in the (x, z) plane $y = -1{\cdot}25D$ ($L/D = 5$, Re = 92).

According to this model, these data can be fitted with a law $\mathrm{Re}_c = A + B(L'/D)^{-2}$ having two adjustable parameters, A and B. The result of such a least-squares fit gives $A \approx 46{\cdot}1$ and $B \approx 252{\cdot}4$. The value of 46·1 for A is very close to the known value, approximately 47, for the critical Reynolds number Re_∞ for the transition to parallel shedding ($L'/D \to \infty$). From parameter B, we can evaluate the diffusive coupling constant μ_r of the model to be approximately 5.1ν which is of the same order as the value $\mu_r = 10(\pm 4)\nu$ obtained by Leweke & Provansal (1994). Let us note that, for this evaluation, we need the value of the linear growth rate $\sigma_r = k(\nu/D^2)(\mathrm{Re} - \mathrm{Re}_\infty)$. In the case of circular cylinders the value $k = 0{\cdot}2$ is valid for aspect ratio as low as 6, but differs to the one obtained for the sphere for which the measurements of Ormières (1999) give $k \approx 1$.

5. CONCLUSIONS AND PERSPECTIVES

The transition from steady to unsteady flow has been studied experimentally in the wake of nine bluff bodies from a circular cylinder to a sphere. The critical Reynolds numbers have been deduced from measurements of oscillation amplitude and have been shown to follow a Landau-type bifurcation in each case. The curves of variation of frequency with the Reynolds number follow a continuous envelope for all nine cylinders. Whereas the visualizations of Zdravkovich *et al.* (1989) show a symmetry change for an aspect ratio around 3,

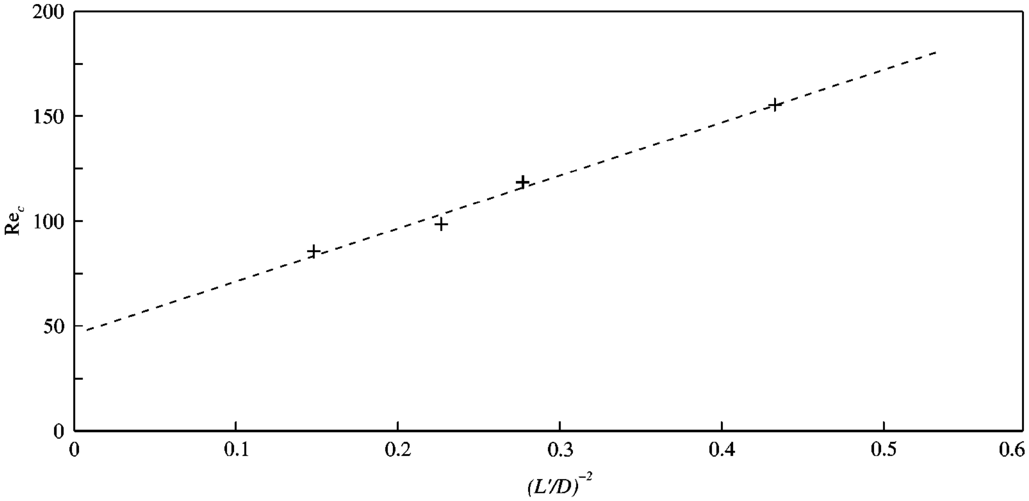

Figure 6. Critical Reynolds number as a function of $(L'/D)^{-2}$, L' is the effective length of the oscillating mode as defined in the text; - - -, fit with the formula $\mathrm{Re}_c = A + B(L'/D)^{-2}$.

such a behaviour is not observed in our quantitative results. Measurements of the amplitude evolution in the spanwise direction have been performed, allowing an effective length of the oscillating modes to be deduced, which is shorter than the geometric cylinder length. Finally, the critical Reynolds number has been plotted as a function of the effective aspect ratio in agreement with the prediction of the Ginzburg–Landau model.

Recently, Owen *et al.* (2000) have been able to suppress the von Kármán vortex shedding and to reduce the drag using a sinuous cylinder, namely a circular cylinder with a constant diameter and a sinuous axis. In such a configuration, the visualizations reveal vortical structures similar to the characteristic connected vortex loops observed behind a sphere or short free-end cylinders. Preliminary measurements have been undertaken with a sinuous cylinder with a spanwise wavelength of 5·6D and a peak-to-peak amplitude of 2D. A critical Reynolds number of 104 has been found. From the results of Figure 3, this value would correspond to a free-end cylinder of aspect ratio between 3 and 4. Moreover, the variation of the frequency measured in the wake of this sinuous cylinder fits perfectly to the continuous law (Figure 4) obtained in the present study, in this range of aspect ratio. In the future, our objective is to compare simultaneous measurements of amplitude, phase and frequency with the case of free-end cylinders. It would be worthwhile to check if a long sinuous cylinder behaves like a collection of free-end cylinder-type oscillators.

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